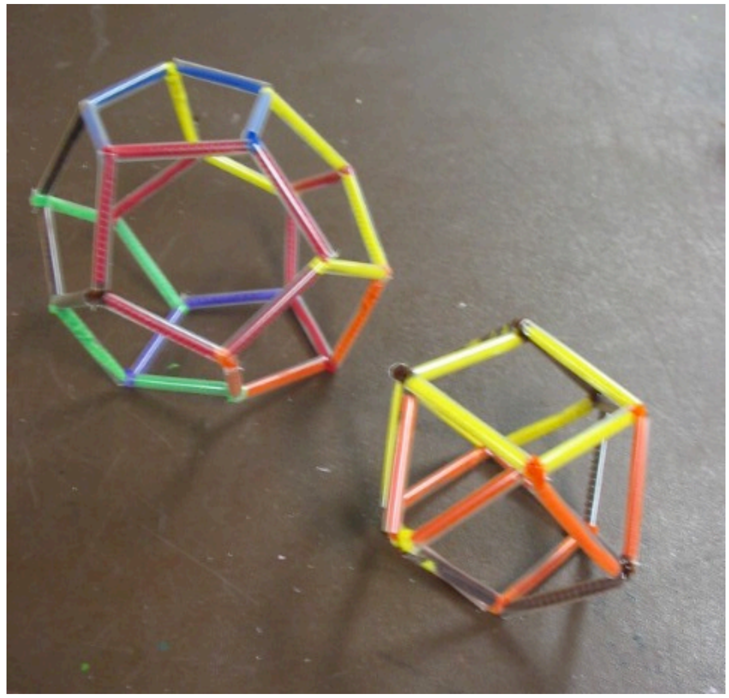
Geometry Construction

Polyhedral playthings



Focus: Math

Parts:

Lots	Straw pieces of identical length – 30 or so for each large polyhedron. Transparent straws show colors nicely.
Lots	Pipe cleaners – 10 or so for each polyhedron.
	Colored string to hang them

Extra Tools:

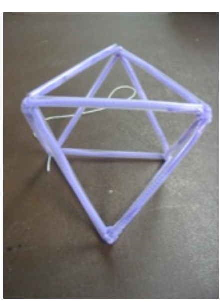
No tool box necessary, but bring: Side cutters Scissors Needle nose pliers

Concepts:

- Only certain combinations of two-dimensional shapes make a polyhedron (three-dimensional shape).
- Polyhedrons with the same shape on each side are called the Platonic Solids. There are only five of them.

Three are made with only triangles:

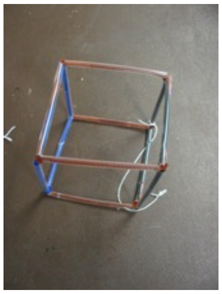
The tetrahedron has 4 vertices.



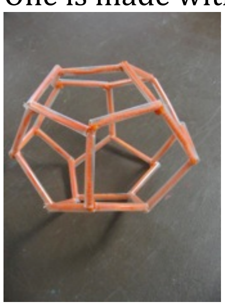
The octahedron has 6 vertices.


The icosahedron has 12 vertices.

One is made with only squares:

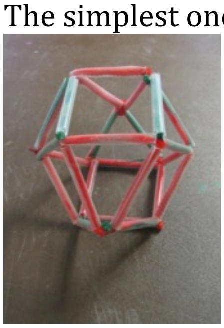

The cube (or hexahedron) has 8 vertices.

One is made with pentagons:


The dodecahedron has 20 vertices.

There are also 13 Archimedean Solids, which employ more than one shape.

The simplest one is made with squares and triangle.


The cuboctahedron has 12 vertices.

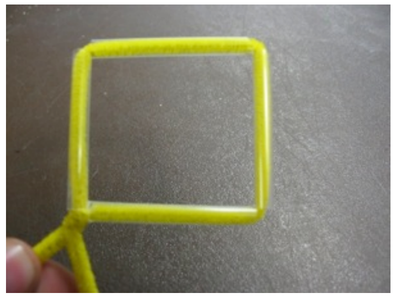
A familiar one is often seen on soccer balls.

The truncated icosahedron has 60 vertices.

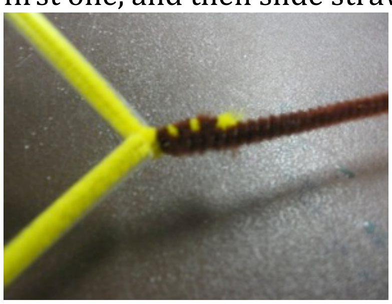
The straws that come together in these projects always make several angles. The sum of these angles is less than 360 degrees, because it is not on a flat surface.

How we build it:

Several basic techniques can be used to construct the polyhedra. They all begin by sliding the pieces of straw onto a pipe cleaner. Close the loop when it has the number of sides you are looking for.

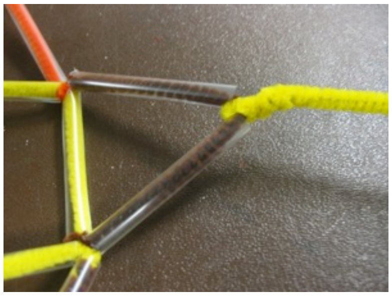
There are at least three ways to proceed. The first is to twist another pipe cleaner to the tail end of the first one, and then slide straw pieces onto it.

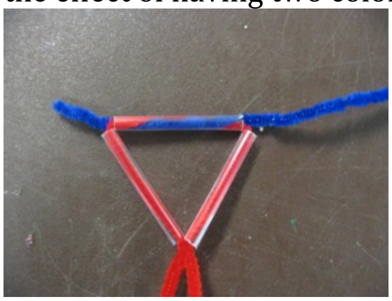
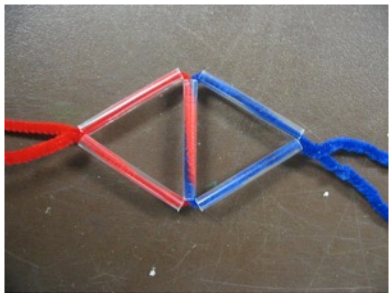
Loop it back around to another vertex and twist it around twice. Add more in the same manner.

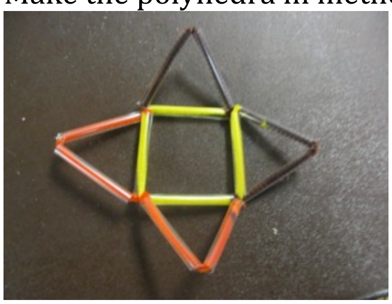
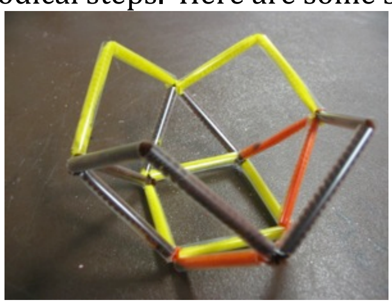
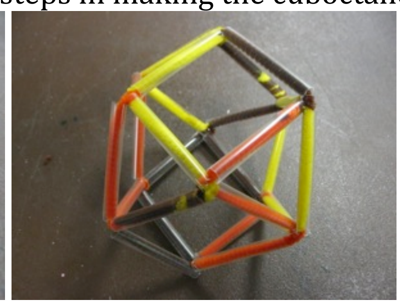
A second way is to twist a new pipe cleaner directly around a vertex. Two twists are enough. The first straw you thread on the pipe cleaner will cover the tail end. Thread the number of straws you wish and loop this pipe cleaner around to twist onto another vertex.

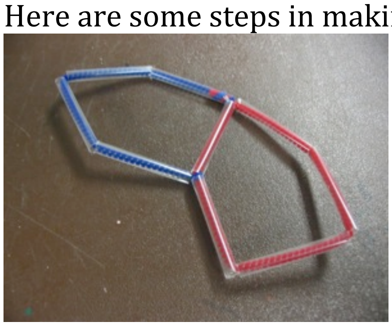
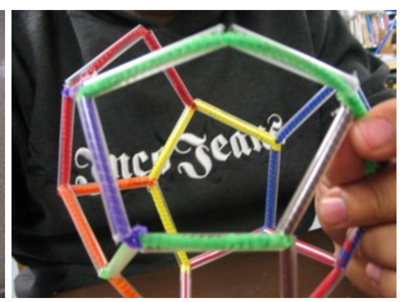
A third way is to thread a new pipe cleaner through a straw that is already in your structure. This gives the effect of having two colors within the same straw.

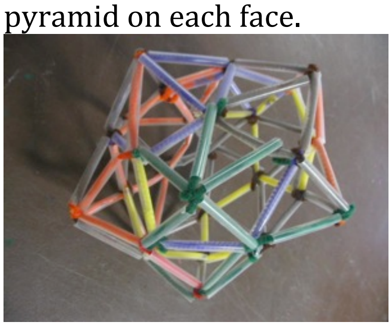
Make the polyhedra in methodical steps. Here are some steps in making the cuboctahedron.

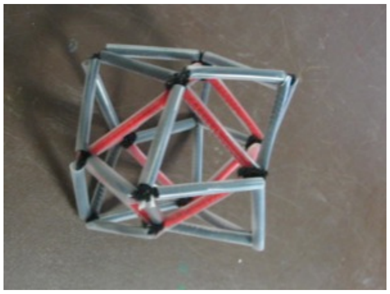
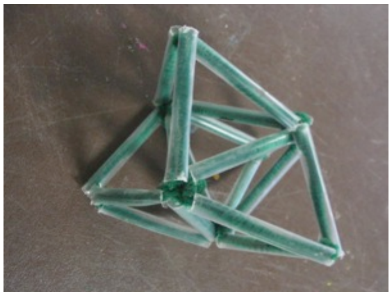
Here are some steps in making the dodecahedron.

One additional variation that can be done is to construct "stellations." This means making a point or pyramid on each face.



Stellated dodecahedron


Stellated cube

Stellated tetrahedron

Two numbers are most useful for constructing each polyhedron:
#1 The number of sides – that is, the shape – of the faces. For example, each face of the dodecahedron is a pentagon; the faces of the cuboctahedron are either triangles or squares.
#2 The number of edges (straws) coming together at each vertex. For example, each vertex of the dodecahedron and the cuboctahedron joins three edges.
As you build the polyhedra, you make decisions about where each pipe cleaner will be joined by considering these numbers over and over. Finding exceptions to these numbers is how to perform troubleshooting. For example, if you are having trouble building the dodecahedron and you find a vertex with four edges coming together, or a face with six sides, you have found a mistake.

Focus Questions:

1. Which of these polyhedra are stable when you push on them, and which bend?
2. What would happen if you used straws of different lengths?
3. How could you make exactly the same shape only bigger?
4. How many faces and edges does the dodecahedron have? The cuboctahedron?

Elaboration:

The fascinating topic of three-dimensional geometry is often overlooked in math class, but we make use of it daily. Vehicles, buildings and containers are all designed using the principles of 3-D geometry.

Counting faces, vertices and edges on polyhedra will give you a feel for the symmetry of each polyhedron. It can be a trick just to find a method to count on the more complex ones. This is a common issue in science and math: Counting is difficult! You will find it difficult to count faces because a face is only space. You can use a marker for edges and vertices. For faces, tape may come in handy. You can make a graph showing the numbers of vertices, edges and faces for each polyhedron.

The polyhedra with triangles are stable, while the cube and the dodecahedron fold when you push on them. Watch the faces when you push on them and you can see why this is true: to squash a triangle, you need to break a side, whereas to squash a square, you just need to bend the vertices.

If you increase the length of your straw sections, you will create the same shape of a greater size. A shape with all sides the same is called a regular polygon. The polyhedra here are special because they are created with regular polygons and are very symmetric. Using straws of different lengths will give you "irregular polygons," and this opens up many other possibilities. Most geodesic domes, for one example, require different edge lengths.

Links to k-12 California Content Standards:

Grades k-8 Standard Set Investigation and Experimentation

Scientific progress is made by asking meaningful questions and conducting careful investigations. As a basis for understanding this concept and addressing the content in the other strands, students should develop their own questions and perform investigations.

Grades k-12 Mathematical Reasoning:

1.0 Students make decisions about how to approach problems:
1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:
2.1 Use estimation to verify the reasonableness of calculated results.
2.2 Apply strategies and results from simpler problems to more complex problems.
2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

3.0 Students move beyond a particular problem by generalizing to other situations:
3.1 Evaluate the reasonableness of the solution in the context of the original situation.
3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
3.3 Develop generalizations of the results obtained and apply them in other circumstances.